Every Student Counts

Professional Development Guide
High School Level

Year 1 - Day 1

Every Student Counts – High School Facilitator Guide
DAY 1 of Year 1

• **Algebra Focus:** Functions (from NCTM Algebra Goal: Patterns, Relations, and Functions)
• **Teaching Focus:** Communication (from NCTM Process Standards)
• **Overall Teaching Focus:** Teaching and learning mathematics through problem solving

TM = Teaching Master
● = materials or equipment

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
Introduction	Overview, expectations, introductions	20	**TM-1** Year 1Outline Chart handout
1. Beliefs About High School Algebra	Small group brainstorming on beliefs of stakeholders about high school algebra	25	**TM-2a-c** Beliefs & Concerns Table handout & overhead • Chart paper, pens, tape for each group
2. Overview of Algebra and Teaching Algebra	Brief presentation of NCTM Algebra and Communications Standards and recent historical conceptions of algebra; view video and discuss	55	• Algebra Overview PowerPoint **TM-3a-b** Algebra Overview handout **TM-4a-b** Commun. Standard overhead **TM-5** *Algebra For All* video and • Video Discussion overhead & handout
3. Algebra Problem-based Instructional Task One	Small groups work on Growing Dots activity, followed by discussion highlighting multiple strategies and mathematical analysis	25	**TM-6** Growing Dots Activity handout **TM-6a** Task – Growing Dots 1
3 cont. Class-room Teaching Video of Growing Dots Activity	View and analyze a classroom video of the Growing Dots activity	25	• Learning & Teaching Linear Functions CD-ROM **TM-7** Hiebert/Wearne Chapter in the Problem Solving Book-overhead **TM-7b** Discussion overhead & handout • Other CD-ROM Resources
4. Algebra Problem-Based Instructional Task Two	Small groups work on Recursive View of Some Common Functions activity, followed by discussion	40	**TM-8** Recursive View of Some Common Functions Activity handout **TM-9a-b** Functions overheads & handout
5. Algebra Problem-Based Instructional Task Three	Small groups work on Trout Population activity, followed by discussion	45	**TM-10** Trout Population Activity handout • Graphing calculators
6. Concerns About Changing High School Algebra	Small groups revisit their Beliefs lists and discuss possible concerns of stakeholders	15	**TM-2a-b** Beliefs & Concerns Table handout & overhead
7. NCTM Communica-tion Standard	Look back at the day's activities through the lens of the Communications Standard		**TM-4a-b** Commun. Standard overhead **TM-11** Reading Assn for Day 2 • Chart paper and markers
Teams Plan	Teams plan action steps	10	None

TM-1 Year 1 Outline

Content Focus: Algebra
Teaching Focus: Teaching Mathematics through Problem Solving

	Day 1	Day 2	Day 3	Day 4	Day 5
Algebra Theme (NCTM Content Standards)	Functions	Algebraic Symbols	Mathematical Models	Linearity	Rate of Change
Teaching Theme (NCTM Process Standards)	Commun-ication	Reasoning and Proof	Problem Solving	Connections	Representation
Mathematical Activities	Investigating functions, especially from a recursive point of view	Making sense of algebraic symbols and mathematical proof	Using mathematical modeling to solve problems in real-world contexts	Investigating linearity in various mathematical settings	Investigating rate of change as a fundamental theme in mathematics
Technology (Graphing calculators each day)	Applets Spreadsheets	Applets Computer Algebra Systems	Spreadsheets	Applets	Calculator-Based Data Collection Devices

Reading assignments each day will include: (1) the Process Standard for the next day, (2) the particular Algebra Standard Goal for the current day, and (3) related research and teaching readings.

Year 1 – Day 1 Agenda

1. Welcome, Introductions, and Overview

2. Warm Up
   Sample of Math in the Movies

3. Beliefs about High School Algebra among Stakeholders
   Small Group Brainstorming and Discussion

4. Overview of Algebra and Teaching Algebra
   Presentation, Brainstorming, Video, and Discussion

5. Initial Task – Growing Dots:
   Activity and Classroom Video

6. Problem-Based Instructional Task 1 – Recursive View of Some Common Functions

7. Problem-Based Instructional Task 2 – Analyzing a Trout Population

8. Concerns about Changing High School Algebra among Stakeholders
   Revisit Beliefs and Discuss Concerns

9. Looking Back at the Day through the Lens of the Communication Standard

10. Closure, Summary, Assignments

Reading Assignment for Day 1

1. Algebra is the overall content focus for the Every Student Counts program.

   - Read "Algebra" for Grades Pre-K – 12 in *Principles and Standards for School Mathematics*, pages 37 - 40.

   - This reading provides a good introduction to the program's perspective on algebra.

   - Reflection Question: How is the view of algebra presented in this reading similar to and different from the algebra that you have taught or learned?

2. Teaching mathematics through problem solving, introduced by the following chapter, is the overall teaching focus for the Every Students Counts program.

   - Read Chapter 1, "Developing Understanding through Problem Solving" by Hiebert & Wearne in *Teaching Mathematics through Problem Solving Grades 6 - 12*. Pages 3 – 13.

   - For Day 1, be prepared to discuss the components of a classroom that is designed to promote understanding.

   - Reflection Question: Briefly describe what it means to "teach mathematics through problem solving." How does this approach develop robust student understanding of mathematics?

3. Communication will be the teaching theme for Day 1.

   - Read "Communication" for Grades Pre-K – 12 in *Principles and Standards for School Mathematics*, pages 60 - 63.

   - Read "Communication" for Grades 9 – 12 in *Principles and Standards for School Mathematics*, pages 348 - 352.

   - Be prepared to analyze the Day 1 activities in terms of the four goals of the Communication standard.

   - Reflection Question: Will "teaching through problem solving," as described in Reading #2, promote the kind of communication described in NCTM's *Principles and Standards*? Explain.

Activity 1 – Beliefs about High School Algebra

Overview and Rationale

This small group discussion is meant to identify typical beliefs of important stakeholders about high school algebra. The results are intended to provide a description of "what is" as a prelude to considering a new vision of high school algebra.

Connections to Other Activities and the Whole Day

This activity starts the day and is a companion of the Concerns activity at the end of the day. The intent is that the present state of beliefs are put on the table at the beginning, activities in between begin to describe the envisioned problem-based algebra content and teaching approach, and then the same set of stakeholders are reconsidered in terms of identifying their likely concerns about moving toward the problem-based approach in their local schools.

How To Carry Out the Activity

- **Introduction**
  Briefly introduce the activity by mentioning the points above.
- **Grouping**
  Set up groups of 3-5 people. Groups are within-area so the discussion can focus specifically on the same locale.
- **Discussion Method**
  Each group considers two stakeholders. They summarize on chart paper the beliefs they generate and give a brief group report to the whole group at the end. All charts are taped to the wall for viewing during the rest of the day.
- **Time**
  Allow 15 minutes to introduce the activity and for groups to discuss the beliefs. Allow 10 minutes for group reports.

Materials

- **TM-2** Beliefs About High School Algebra handout
- Chart paper, pens, and tape for each group

TM 2a

Beliefs about High School Algebra

- Each group focuses on two stakeholders.

- Write on large sheet likely beliefs of each stakeholder about high school algebra (15 minutes).

- Report your group's results to the whole group (10 minutes).

- We will collect, summarize, distribute and revisit on Day 2.

TM 2b

Role	Concerns About Changing HS Algebra
• A student	
• A classroom teacher	
• A principal	
• A counselor	
• A superintendent	
• A district math curriculum coordinator	
• A school board member	
• A parent of a student	
• A business person in the community	
• A teacher in a college/ university math department	

TM 2c (4 pages)

Beliefs and Concerns of Various Stakeholders about High School Algebra

Comments from Workshop Conducted in September 2004

Beliefs	Concerns
A Classroom Teacher	
• Stepping stone to higher math • Special Ed teachers feel inadequate to support student learning • General Ed unsure how to teach Special Ed students • Algebra is not accessible to all students • Content – basic how can you not get this? But it is so important at the same time. Some don't think Algebra has changed? How can it change? • Teaching – building block, gateway to other classes and careers • Special Ed – Too abstract for student with special needs • Discrepancy of Algebra: is it for everyone or only a few? • Very important, foundational students will need to build and extend to understand "higher" math. • Rules, Algorithms • For bright or smart kids and will be hard for some • Students will reach a developmental stage of thinking to be able to formalize • Teach how they were taught • Given curriculum/did not choose • Some want all students, some don't (college prep only) • Teach procedure not application	• Time • Dollars for technology and equipment • Lack of professional development • Noisy rooms/classes • Change • Staff dump – ongoing • Philosophy • Attitude • Effort • Time • Materials & supplies, adequate? • Classroom management • Professional development • How do I learn how to teach this way? • How do I learn to use the technology? • How quickly do I need to learn? • How will the kids/community accept the change?
A School Administrator	
• All students need Algebra to be proficient on tests • Pressure on how to progress – traditional v. integrated • Too abstract, too difficult for students (a lot due to person's prior experience with algebra) • Let's just cover basics! • Algebra doesn't help with ITED's, so let's get rid of it.	• Training teachers that come in post-implement • Resources • Time for professional development • Understanding • Calculators • Focus on instruction rather than student learning • What will changes do to test scores? • How to budget SD?

• Algebra has no connection to "real life" • Real life looks different to different kids. • Algebra is a "must have" • We need if for good scores on ITED's and SAT's, college prep • I'm a little uncomfortable evaluating that teacher/class • I enjoy the organizational and analytical skills of my algebra teacher, great to head committees • Algebra may be reserved for upper-level track kids • Just a math credit • Do I have qualified teachers? • Budgetary concerns regarding inservice training	• Parental concerns • Do S&B align with curriculum? • How do I explain this to parents/community? • How do we pay for technology and training? • How will this impact student achievement? • What do I do with teachers who don't want to change?
A Counselor	
• Only students going to college need algebra • Not all students can do algebra • Do not understand mathematics • Believe algebra should be taught traditionally • Placement/scheduling • Prerequisites • Sorter/gatekeeper • Size of the school matters • Not worried about where kids are placed • Look at requirements to graduate & push them through • No concern about being able to do the work • Other opportunities for all students to succeed • Place students in courses that they can handle • It's required for HS graduation • Needed for college entrance • May be necessary for some populations	• Scheduling problems – how to get every student into algebra, creating new/additional sections • Teacher concerns about placement of certain students • Parental concerns about students signing up for algebra • How will test scores (ACT, etc.) be affected? • Convincing students they can do it • Understanding how algebra has changed • They would promote algebra if they understood the approach, would meet the needs of divers learners • Credit & grading issues for group work • Forced to place kids by ability • Material (content) less rigorous, not prepared for college • Less teachers available to teach wide range of courses • Students will seek other resources for courses • May be unnecessary for some populations • What to do when students don't pass (when it is a requirement)

A Superintendent or Curriculum Person	
• View as test scores • P.R. for community • May be out of touch • Not aware of class size issues • Hear from disgruntled parents • More curriculum out front and pushing for new ways to teach algebra • Superintendent – not easy to find high school teachers • State & Federal requirements • Want consistency throughout grade levels in text & teaching methods • Picking up on gaps in teaching the standards • Having the prerequisites met to go on to next level or course • It's part of math series of courses • Perception based on data • Alignment of standards/benchmarks with course content	• Professional development cost and time • Information to the public? • How the approach would impact test scores • Support smaller class size for hands on learning • Cost of technology and materials • How will it change how kids perform on district-wide tests? • How will I handle staffing? • What will teachers/parents say? • Not meeting standards/benchmarks • Less data to review • Professional development to implement change • Lack of funds/materials to implement change • Keeping up with research-based practices • ACT scores reflecting required math courses (or lack of required algebra course) • ITED may/may not be algebra-focused
A School Board Member	
• Algebra is the same as it was when I was in high school • Algebra is very different than it used to be • Algebra is a gatekeeper course • Algebra is a math offering • Algebra for higher performing students (college prep) • Not a specific requirement for graduation • Will ask "should algebra be taught as a course at Middle School? • Looks at algebra as a course, not a concept-content • Kids need to know basics first	• Will this hurt test scores? • Cost (initial and ongoing, recurring costs) • Will parents call me about it? • Dumbed down? • Ready for college? • Cost of technology • Is it broken? Why fix it? • How to promote algebra for <u>ALL</u> • If students are taught at 8^{th} grade, there needs to be courses available at the HS (11^{th} & 12^{th}) level too • How to clarify a common definition of algebra • How to convince them that basics is more than +, -, x

A Parent of a Student	
• My kid needs algebra, but I'm not so sure all kids need it • Algebra is a one year event • I don't use algebra • Necessary evil • "C" grade is acceptable • College-bound course • Males do better • Can't help with homework • Manipulating symbols • No practical applications • "A" student is smart • Parents accept that some students have more difficulty with math than with reading • Algebra is a course, not content	• Cost of technology • How do I help my child? • Will my child be at a disadvantage? • My kid is the guinea pig? • Where is the computation (symbol manipulation, etc.)? • Application • Show them • Family Math night • Too slow – movement in curriculum • Be ready for • Different • Calculators = cheating? • Is algebra relevant? • What are the basics-teach those first
A Business Person in the Community	
• Awareness, target #'s, projections • Field, you are in (construction, actuary, machinist, tire retreading) • Function, if then result of input-output • Doctors, meds, body weight, nursing, anesthesiologist • Level along ladder • Perceived notion that the school has a strong math program or a weak one • Graduates from HS should have basic consumer math skills • See math as a big concept – not broken down into "algebra"	• People might worry that learning would be too spotty • Text has page-by-page curriculum. • Some aren't convinced new curriculum is without holes. • Like "new math," been there done that • It was good enough for me • Like whole language idea, had to go back to phonics • Will students be able to transfer skills to real-life problems & applications • Will they find a use for it?
A Teacher in a College/University Math Department	
• Students should come with strong understanding of algebra • Students know formulas • Exposure to advanced algebra • Students come from a very structured algebra class in HS • Students come to college with no anxiety about math • Teachers/professor may lack connections w/real world • Think students should come w/readiness to learn • Lack empathy w/students who struggle w/math	• Focus • Old school separate algebra out • Those caught in NCTM for blend of algebra and other areas • Foundation for abstract thinking for generalizations • Tech link • Won't have basic skills • Won't be able to make connections

Activity 2 – Overview of Algebra and Teaching Algebra

Overview and Rationale

Viewing and discussing the *Algebra for All* video provides a vision of algebra and (somewhat) of teaching and learning that are the themes of the *Every Student Counts* program.

Connection to Other Activities and the Whole Day

This activity provides a general introduction to the algebra content and teaching vision of the NCTM Standards. The activities that follow later in the day provide specific examples of the broad concepts in this introductory activity.

How To Carry Out the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction**
  Give a quick review of the Historical Conceptions of Algebra chart, the Communications Standard, and the Algebra Standard, using the Algebra Overview PowerPoint. Introduce the video as presenting the NCTM vision of algebra and how it is taught and learned.
- **Video Discussion Method**
  Provide participants with a focus before viewing the video, namely, they should compare what they see in the video to (1) the Algebra and Communication Standards Day 1 readings from *PSSM*, (2) the historical conceptions of algebra, and (3) their own experience teaching and learning algebra. View the *Algebra for All* video. Discuss in small groups. Have three groups report out, each based on one of the three above foci.
- **Closure**
  This video was made before *PSSM*. Three teachers in the video – Allan Bellman, Cheryl Hedden, and Dan Langbauer – are authors of teacher stories in the *Teaching Mathematics Through Problem Solving* book. So far, the Beliefs activity has given us an idea of what we are up against, and this video provides a general picture of where we would like to be.
- **Time**
  Allow 10 minutes to introduce the activity, 30 minutes to view the video, 10 minutes for group discussion, and 5 minutes for group reports.

Materials

- *Algebra for All* video (PBS Mathline, High School Math Project: Focus on Algebra, Tape One, http://teacher.shop.pbs.org/product/index.jsp?productId=1826405)
- **TM-3a-b** Algebra Overview PowerPoint and Handout
- **TM-4** Communications Standard main points overhead
- **TM-5** Overhead containing the three discussion foci

TM-3a

Algebra Overview (Text of Slides)

What is algebra?
What are the big ideas of algebra?
What should students be able to do in algebra?

From your reading:
The National Council of Teachers of Mathematics
Principles and Standards for School Mathematics

ALGEBRA …

Patterns, relations, functions

- Understand patterns, relations, and functions

Algebraic symbols and operations

•Represent and analyze mathematical situations and structures using algebraic symbols

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Mathematical models of quantitative relationships

•Use mathematical models to represent and understand quantitative relationships

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Change

•Analyze change in various contexts

Is this how you, your students, or parents would describe algebra?

Or, perhaps they would describe things like …

Solving equations

For example, “solve for x” in

$3x + 7 = 8$

or

$x^2 + 5x + 6 = 0$

Algebraic systems and algebraic properties

Like groups, rings, and fields;
or like the commutative property:
$a + b = b + a$

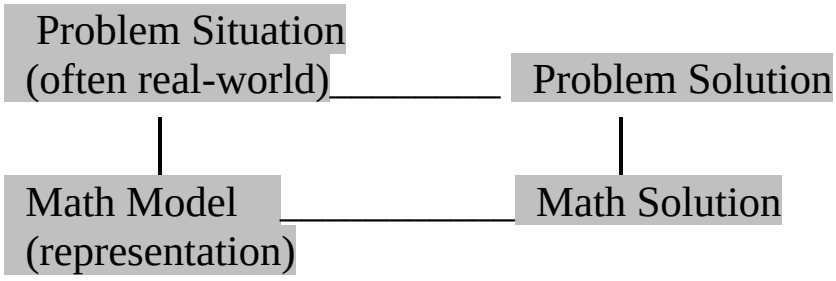
Operating on algebraic expressions ("symbolic manipulation")

Simplify
Expand
Factor
(and Solve)

Story Problems

Let x = ….
then …

Mathematical Modeling



Modeling

Given this problem situation (real world) ….

- Represent the situation using mathematics (i.e., build a mathematical model).
- Operate on the mathematical model to get a mathematical solution
- Interpret the mathematical solution to get a real-world solution.
- Compare to original problem; evaluate, adjust, finalize, or perhaps start cycle over again.

Functions

f(x) = ….
graphs
tables
symbolic rules

Conclusions

•These classroom-based descriptions of algebra are compatible with NCTM descriptions
•Students need this broader and deeper understanding of algebra
•Can be summarized historically …

Recent Historical Conceptions of Algebra

1950s - now

Movement	Period	Algebra is (about):	Focus on:
Theory of Equations	1950s	Solving equations	Techniques for solving equations
New Math	1960s	Structure (re: abstract alg)	Alg systems and properties
Back to Basics	1970s	Generalized arithmetic	Alg expressions-simp, exp, factor
Problem Solving	1980s	Problem solving	Using alg to model and solve
1989 NCTM Standards	1990s	Functions (re: analysis) & PS	Patterns of chg, quant. relations.
2000 NCTM Standards	2000s	Synthesis of the Above	Balanced approach

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Video Discussion Points

- Compare video with Algebra and Communication reading from *PSSM*
- Compare video to historical conceptions chart
- Compare to your own views and experiences about teaching and learning algebra (re: reflection question from the reading assignment)

TM 3b
Algebra Overview

What is algebra?
What are the big ideas of algebra?

NCTM Algebra Standard

Algebra is (about):

- patterns, relations, functions
- algebraic symbols and operations
- mathematical models for quantitative relationships
- change

Recent Historical Conceptions of Algebra

Movement	Approx. Period	Algebra is (about):	Focus on:
Theory of Equations	1950s	Solving Equations	techniques for solving equations
New Math	1960s	Structure (re: abstract algebra)	algebraic systems and properties
Back to Basics	1970s	Generalized Arithmetic	algebraic expressions - simplify, expand, factor
Problem Solving	1980s	Problem Solving	using algebra to model and solve problems
1989 NCTM Standards-Based Reform	1990s	Functions (re: real analysis) and Problem Solving	patterns of change, quantitative relationships
2000 NCTM Standards-Based Reform	2000s	Some of All of the Above	balanced approach

TM4a

Algebra Standard

Instructional programs should enable all students to—

- Understand patterns, relations, and functions

- Represent and analyze mathematical situations and structures using algebraic symbols

- Use mathematical models to represent and understand quantitative relationships

- Analyze change in various contexts

TM4b

NCTM Communication Standard

Instructional programs should enable all students to—

- Organize and consolidate their mathematical thinking through communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

- Analyze and evaluate the mathematical thinking and strategies of others

- Use the language of mathematics to express mathematical ideas precisely

TM5

Algebra for All
Discussion of Video

- Compare video to historical conceptions of algebra

- Compare video to the Algebra and Communications reading assignment in *PSSM*

- Compare video with your own view and experience teaching and learning high school algebra

TM5 continued

Algebra for All Video

Discussion Comments from Workshop Conducted in September 2004

Video and NCTM Algebra Standard

- Very compatible
- All the process standards
- More than just skills, skills, skills
- Can discover the skills
- Skills and concepts
- Using math models, setting up and using
- Consistent views
- Student centered
- Use of technology
- Interest and motivation
- Applied problems

Video and Historical Conceptions

- Math classes used to be almost exclusively skill driven, now many of those skills are built into calculators, so we don't need to do so much skill work ….
- More student-centered, rather than teacher-centered
- More of an emphasis on understanding, rather than on just doing procedures
- Assessment is richer, quizzes, tests, monitor group work, student experiments
- Practical applications plus technology allow you to solve richer problems
- "I have a reason why I'm teaching math."
- Lots of science-based applications
- Do math, not just practice procedures
- More focus on problem solving
- Moving from the 1980s on
- Function emphasis
- Applications, helps meaning

Video and Your Own Experience Teaching and Learning Algebra

- Must communicate with and bring students on board, they may resist the "new approach"
- The "gifted" kids may be vocal resisters, since they mastered the old system
- How to work with larger classes, smaller classrooms
- Getting technology for all may be a barrier
- We may pose complex tasks, but we also often provide so much support that the richness and educational value of the task is reduced
- Frustrating for teachers to make the change, not enough time, resources, training
- New vision is overwhelming
- Will take time and effort to implement
- Student to student interaction

Activity 3: Algebra Problem-Based Instructional Task One
Growing Dots Activity

Overview and Rationale

This activity provides a gentle introduction to functions, the algebra theme for Day 1, through an examination of dot patterns. Participants first work in groups on the Growing Dots task, and then view a video clip of the same task being taught in a highly interactive way to a high school class. This gives them the opportunity to see an example of both the problem-based content and a class in which the mathematics is taught through problem solving.

Connection to Other Activities and the Whole Day

Through the group communication and the interactive teaching on the video clip of a classroom, participants see examples of the Communication process standard in action. The second and third Problem-Based Instructional tasks build mathematically and pedagogically on this one, in particular, by the use of multiple strategies including recursion. This activity also connects to the multiple strategies point in the Hiebert & Wearne reading from the Problem Solving book.

How to Carry Out the Activity

- **Grouping** Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction** Emphasize that this task introduces functions, today's algebra theme. "If you don't see the answer, don't worry. Strategies, right or wrong, are sites for learning. If you see the answer right away, don't blurt it out."
- **Group Problem Solving and Discussion** Each group works on the Growing Dots problem, looking for as many different solution strategies as they can find. The results are discussed within the whole group, as follows: (1) participants describe solution strategies, (2) look for whether they add 4, multiply by 4, think recursively, start at stage 0 or stage 1, and what representations they use.
- **Video Viewing and Discussion** View the two solution segments on the Dots video clip from the CD. Have half the participants in each group (perhaps those with high school math teaching preparation) focus on the student's solution strategies and the other half focus on the teaching strategies. Discuss the video clip from the perspective of these two foci and in light of their own experience working on the task.
- **Closure** Briefly refer to and discuss the main points of the Hiebert/Wearne chapter: allow mathematics to be problematic, focus on solution methods, and tell the right things at the right time.
- **Time** Allow at most 5 minutes for a brief introduction, 10 minutes for groups to work on the Growing Dots task, 10 minutes to discuss their solutions and strategies in the whole group, 10 minutes to view the video clip, 10 minutes to discuss the video clip, and 5 minutes for closure.

Continues on next page.

Materials

- **TM-6** Growing Dots task
- Video clip on CD-ROM of the *Growing Dots* task in a classroom (*Learning and Teaching Linear Functions CD*, Heinemann, 2004)
- **TM-7** Main points from Hiebert & Wearne chapter overhead
- **TM-7b** Discussion of Video on *Growing Dots*, Overhead & Handout
- Further resources about the Growing Dots task, and other related tasks, can be found in the Resources on the CD.

TM6

Growing Dots (textbook lesson)

TM 7

Growing Dots Activity - Discussion Related To: *Designing Classrooms to Promote Understanding*

(Hiebert and Wearne chapter in the Problem Solving book)

How did your work and our discussion of the Growing Dots activity illustrate the following three points that characterize classrooms that promote understanding?

1. Allow mathematics to be problematic for students.

2. Focus on methods (strategies).

3. Tell the right things at the right time.

TM 7a continues
Growing Dots Activity: Classroom Teaching Video
Discussion Comments from Workshop Conducted 9/04

Algebra and strategies:

- 4 + x versus 4x + 1, x is PREVIOUS versus x is time
- Teacher needs to be alert about this important point
- Each leg grows by one, for a total of 4 new dots since 4 legs
- James was getting at recursive
- Teacher might have done a bit more with this after identifying it
- Comparing and using the two representations – importance of the variable:
  - Y=4x + 1
  - Y = x + 4, really Previous + 4
- Starting value
- What does variable mean, and how it connects to the context

Teaching:

- Teacher allowed mathematics to be problematic
- Mainly worked with just a couple students
- Teachers need to learn to process student results
- Compare and contrast and analyze different methods used
- Students might have benefited from having more strategy discussion among themselves
- The "quarter" example crystallized the confusion about just looking at what added and not where started
- "correct me if I'm wrong" builds good relationship with students and problem solving
- Discussion about "x" was interesting – what does it represent, and keep it the same for a given solution
- Teacher used good questioning, e.g., redirecting students' thinking
- Problems solving approach implies a different classroom environment
- Culture of safe learning environment
- Safe to make mistakes, it can be scary to make conjectures
- Good questioning skills
- Good facilitation skills
- Kept kids going investigating the problem

Activity 4: Algebra Problem-Based Instructional Task 2
A Recursive View of Some Common Functions

Overview and Rationale

This activity continues the development of functions, the algebra theme for Day 1, with particular focus on the idea of recursion. Recursion is the process of describing the present state of a system in terms of the previous states(s). The recursive point of view is likely to be unfamiliar to many participants, yet it is accessible, powerful, and recommended in the NCTM Standards. Participants will investigate linear and exponential functions from a recursive point of view.

Connection to Other Activities and the Whole Day

This activity continues the development of all the main themes for Day 1 – functions, communication, and teaching and learning through problem solving. In particular, this activity provides a more in-depth look at the NCTM standard relating to functions.

How To Carry Out the Activity

- **Grouping**
  Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction**
  Introduce the activity as another example of an algebra problem-based instructional task. Don't discuss recursion, linear functions, or exponential discussions in the introduction. Just get the participants going on the investigation. The discussion will come later. This activity is from the upcoming book from NCTM, *Navigating Through Discrete Mathematics in Grades 6 Through 12*.
- **Group Problem Solving**
  Each group works on the handout entitled "Investigation 1: A Recursive View of Some Common Functions."
- **Discussion**
  Have two groups present their completed charts from Part 1 and Part 2, respectively. Discuss these charts. Also, summarize the differences in Parts 1 and 2 between NEXT/NOW equations, y=… equations, tables, and graphs.
- **Closure**
  Be sure to highlight how the NEXT/NOW representation (recursion) is very accessible and very clearly shows the key features of linear functions (slope, constant rate of change, and added constant) and exponential functions (multiplied constant). At this time you can also use overheads to briefly discuss what is a function and summarize the NCTM Algebra Goal dealing with functions, and relate to the activity just completed.

Continues

- **Time**
  Allow 20 minutes for groups to work on the investigation. If progress is slow, you could have some groups work on Part 1 and other groups work on Part 2. However, it is best if everyone does both parts, and in any case a good discussion and comparison of both parts is important. Allow 10 minutes for whole group discussion afterwards. Allow 10 minutes for closure.

Materials

- **TM-8** Recursive View of Some Common Functions activity handout
- **TM-9a** What Is A Function overhead
- **TM-9b** NCTM Algebra Functions Goal overhead
- Handout for the overheads above

TM 8 (5 pages ©2007 NCTM)
Investigation 1: A Recursive View of Some Common Functions

Functions can be represented in several ways, including with tables, graphs, and equations. In this investigation you will examine representations of two fundamental types of functions. Your goal, by the end of the investigation, is to find answers to these questions:

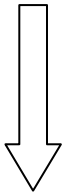
How can each function be represented by two different types of equations?
How does each equation show key properties of the function and its graph?
How do the equations help you see similarities and differences between the two functions?

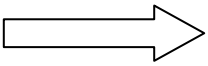
Part 1: Constant Rate of Change

Consider the function table below. Your task is to complete all parts of the chart below, by solving the problems on the next page. Go on to the next page and get started.

x	***y***
0	4
1	7
2	10
3	13
4	16
:	:

Recursive Formula





Sketch graph below:

Explicit Formula

Slope
Constant rate of change
Constant added to get from one y-value to the next

TM 8 continues
Part 1 Problems

1. Before completing any part of the chart on the previous page, examine the given function table and briefly describe any patterns you see in the table.

2. Look down the column of y-values in the table. If NOW is the y-value at a particular place in the table and NEXT is the y-value in the next row down, write an equation using NEXT and NOW that describes the pattern in the y-values. Write this equation in the box next to the down-arrow in the chart on the previous page.

3. Now look across the columns. Write an equation in the form "y = …" that shows the relationship between x and the corresponding y. Write this equation in the box below the right-arrow in the chart on the previous page.

4. What type of function is represented by the table and the equations you have found? Describe the basic characteristics of this function.

5. Sketch a small graph of this function, in the location shown on the previous page. Slope is a fundamental feature of the graph of a linear function. What is the slope of the graph of this function?

6. The slope of a graph of a linear function also represents the constant rate of change of y with respect to x. Describe how this constant rate of change is shown in the table.

7. Examine the equations from Problems 2 and 3.
   - Describe how the slope and constant rate of change are shown in each of the two equations you have written in the boxes in the chart on the previous page.
   - Circle the number in those equations that corresponds to the slope. Draw an arrow from each circled number to the box at the bottom of the page, to show what the number represents.
   - Do you think one equation shows the slope and constant rate of change more clearly than the other? Explain.

8. Review your completed chart. Be prepared to explain all the information on the page to your classmates. In particular, for each item on the chart page be prepared to answer these questions:
   - What is it?
   - How is it determined?
   - What does it mean?
   - How does it connect with other information on the page?

TM 8 continues

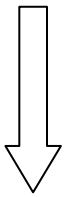
A Recursive View of Some Common Functions

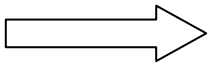
Part 2: Constant Multiplier

Consider the function table below. Your task is to fill in the boxes in the chart below, by solving the problems on the following page. Then you will compare to the work you did in Part 1. Go to the next page and get started.

x	*y*
0	4
1	12
2	36
3	108
4	324
:	:

Recursive Formula





Sketch graph below:

Explicit Formula

Constant multiplier
Non-constant rate of change

TM 8 continues
Part 2 Problems

1. Before completing any part of the chart on the previous page, examine the given function table and briefly describe any patterns you see in the table.

2. Look down the column of y-values. If NOW is the y-value at a particular place in the table and NEXT is the y-value in the next row down, write an equation using NEXT and NOW that describes the pattern in the y-values. Write this equation in the box next to the down-arrow in the chart on the previous page.

3. Now look across the columns. Write an equation in the form "y = …" that shows the relationship between x and the corresponding y. Write this equation in the box below the right-arrow in the chart on the previous page.

4. What type of function is represented by the table and the equations you have found? Describe the basic characteristics of this function.

5. Sketch a small graph of this function, in the location shown on the previous page. Does the graph have a constant slope? Is there a constant rate of change of y with respect to x? Explain.

6. A fundamental characteristic of exponential functions is that there is a constant multiplier (but *not* a constant rate of change of y with respect to x). Describe how the constant multiplier is shown in the table.

7. Examine the equations in Problems 2 and 3.
   - Describe how the constant multiplier is shown in each of the equations that you have written in the boxes in the chart on the previous page.
   - Circle the number in those equations that corresponds to the constant multiplier. Draw an arrow from each circled number to the box at the bottom of the page, to show what the number represents.
   - Do you think one equation shows the constant multiplier more clearly than the other? Explain.

8. Review your completed chart. Be prepared to explain all the information on the page to your classmates. In particular, for each item on the chart page be prepared to answer these questions:
   - What is it?
   - How is it determined?
   - What does it mean?
   - How does it connect with other information on the page?

TM 8 continues

9. **Comparing Part 1 and Part 2**

Compare the NEXT/NOW equations you wrote in Problem 2 of Part 1 and Part 2. (These can be called *recursive formulas*, since they describe one value in terms of the previous value.)

a. Describe how these two NEXT/NOW equations are similar and different.

b. How is this difference shown in the "y = …" equations? (See Problem 3 of Part 1 and Part 2.)

c. How is this difference shown in the graphs of the two functions?

d. How is this difference shown in the tables for the two functions?

10. **Extension**

In this investigation you have explored two recursive formulas:

NEXT = NOW + k

NEXT = NOW x k

Consider a "combined recursive formula:"

NEXT = r NOW + b.

a. Why do you think the phrase *combined recursive formula* is used? Specifically, explain why you think the word "combined" is used.

b. Go to the CD and carry out an applet-based investigation of this equation, using graphs, tables, and y = … equations, in the context of a changing fish population. On the CD, see the investigation called "Trout Pond."

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TM9

What is a FUNCTION?

Relationship between 2 quantities
2 quantities change relative to each other

For example, number of dots increases as time increases (re: Growing Dots Activity)

# of dots is a function of time
n is a function of t
n depends on t
y is a function of x
y depends on x

y = f(x)
x is an input, which yields output y, according to the function f
Each input yields exactly one output

TM9 continued (Overhead version)

NCTM Algebra Standard
Functions Goal

Goal 1

Instructional programs from prekindergarten through grade 12 should enable all students to:

Understand patterns, relations, and functions

In grades 9–12 all students should:

- generalize patterns using explicitly defined and recursively defined functions;
- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- interpret representations of functions of two variables

TM9 continued (Handout version)

NCTM Algebra Standard
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Each input yields exactly one output

Activity 5
Algebra Problem-Based Instructional Task 3
Using Algebra to Analyze a Changing Trout Population

Overview and Rationale

In this activity participants engage in more work with algebra in general and recursion in particular. Participants will use recursion to analyze a changing trout population. Specifically, they will develop a NEXT/NOW equation that models a given population situation, and then use the "next answer" feature of a graphing calculator to numerically analyze the situation. As time permits, they will also analyze the situation graphically and with algebraic closed-form equations.

Connection to Other Activities and the Whole Day

This activity continues the development of all the main themes for Day 1 – functions, communication, and teaching and learning through problem solving. In particular, the development and use of recursion continues. Recursion emerged informally in the Growing Dots activity, then it was explicitly discussed in the previous Recursive Point of View activity, and now recursion will be implemented using technology.

How To Carry Out the Activity

- **Grouping** Groups are formed so that at least one person with secondary mathematics teaching preparation is in each group.
- **Introduction** Introduce the activity as another example of an algebra problem-based instructional task, one in which we apply and refine our growing knowledge of recursion. This activity is adapted from an online applet-based activity on the NCTM Illuminations Web site, *Using Algebra and Discrete Mathematics to Investigate Population Changes in a Trout Pond*. http://illuminations.nctm.org/index_o.aspx?id=142
- **Whole Group Launch** Lead a discussion of the "Launch and Overview" with the whole group. You may also decide to work as a whole group on Problems 1-3 to make sure everyone understands the NEXT/NOW equation and how to use the calculator.
- **Small Group Problem Solving** Each group works on the handout entitled "Using Algebra to Analyze a Changing Trout Population." For the "What if …" analysis in Problems 7-9, you might assign different groups to investigate the effect of changing the three different parameters – initial population, restock amount, and annual decrease rate. Note that Problem 11 is optional and requires access to the NCTM Illuminations Web site.
- **Whole Group Discussion** Briefly discuss the results of Problems 4, 5, and 6. Summarize the results of checking conjectures in Problem 8. That is, summarize the effect of doubling initial population, restock amount, and decrease rate, respectively. More generally, summarize the patterns found in Problem 9. That is: (i) changing the initial population results in no change in the long-term population, (ii) changing the restock amount results in proportional change, and (iii) changing the decrease rate results in inverse proportional change.

- **Closure** Highlight how the NEXT/NOW representation (recursion) and technology have provided *access* to this problem, whereas otherwise it is only accessible to advanced high school algebra classes. Note that this is only a numerical analysis, and that a further graphical analysis can be done, and further symbolic algebraic analysis is needed to verify the numeric patterns that we have noticed. You can point out that this activity involves reasoning and modeling, which are two themes that will be emphasized in Days 3 and 4, respectively.
- **Time** Allow 30 minutes for groups to work on the activity. If progress is slow, you could have different groups analyze different parameters in Problems 7-9. Allow 10 minutes for whole group discussion afterwards and 5 minutes for closure.

Materials

- **TM-10** Using Algebra to Analyze a Changing Trout Population activity handout
- Graphing calculators (TI-83 or better)

TM 10
Using Algebra to Analyze a Changing Trout Population

Summary and Adaptation
From the NCTM Illuminations Web Site
"Using Algebra and Discrete Mathematics to Investigate Population Changes in a Trout Pond"
http://illuminations.nctm.org/index_o.aspx?id=142

Investigation Summary
This investigation illustrates the use of iteration, recursion, and algebra to model and analyze a changing fish population. Graphs, equations, tables, and technological tools are used to investigate the effect of varying parameters on the long-term population.
Part 1: Launch and Overview
Part 2: Numerical Analysis
Part 3: Graphical Analysis
Part 4: Symbolic Analysis

Launch and Overview

A fishing pond is stocked with trout. You are to manage the trout population in the pond. In order to mathematically model this situation, the following simplifying assumptions are made:

- There are currently 3000 trout in the pond.
- Regardless of restocking, the population decreases by 20% each year due to the combined effect of all causes, including natural deaths and trout being caught.
- 1000 trout are added at the end of each year.

Do you think the population will grow without bound, level off, oscillate, or die out? Explain why you think your conjecture about long-term population is reasonable.

Part I: Numerical Analysis

Your goal is to investigate the long-term population of trout in the pond. First, you will do a couple of computations "by-hand" to get a feel for how the population changes according to the assumptions above. Next, you will write an equation that represents the situation. Then, you will use a calculator or spreadsheet to numerically investigate the long-term population. Finally, you will find and describe patterns of change in the long-term population.

1. Using the assumptions in the box above:
    - Find the size of the trout population one year from now.

- Find the population two years from now.

2. Let the word NEXT represent the population next year, and NOW represent the population this year. Write an equation using NEXT and NOW that represents the assumptions about the trout population given above.

3. Use your NEXT/NOW equation from Problem 2 and the ANS and ENTER keys on a graphing calculator to find the population many years from now. (Or use a spreadsheet.)
   - What will be the population 50 years from now?
   - Was your conjecture about the long-term population correct?

4. When does the trout population reach 5000? Does the mathematical model ever actually yield 5000? Explain.

5. Does the trout population change faster around year 5 or around year 25? How can you tell?

6. Explain, in mathematical terms and in terms of the fishing pond ecology, why this long-term population is reasonable

What if…?
Find out what happens when the assumptions change.

There are three key factors in this problem - the initial population, the annual restocking amount, and the annual population decrease rate.

7. Consider the three questions below. Make conjectures for the answers.
   a. If the initial population doubles, what will happen to the long-term population?
   b. If the annual restocking amount doubles, what will happen to the long-term population?
   c. If the annual population decrease rate doubles, what will happen to the long-term population?

8. Use a calculator or spreadsheet to test your conjectures for the three questions above. Any surprises? Any patterns?

9. Systematically investigate the effect that other changes in these three factors will have on the long-term population. Keep track of the results of your investigations. Describe any patterns you see.

10. Suppose the annual decrease rate is 20% and the initial population is 3000, as in the original situation. But now suppose you want to change the restocking amount so that the long-term population will be 7500. What restocking amount should you use?

11. (**Optional**) Continue analyzing this situation using graphs and closed-form algebraic equations at: http://illuminations.nctm.org/index_o.aspx?id=142

Activity 6
Concerns About Changing High School Algebra

Overview and Rationale

This small group discussion is meant to identify typical concerns of important stakeholders about high school algebra. The results are intended to provide a description of what some of the local issues and barriers are likely to be if high school algebra is to change in the direction of the ESC/NCTM vision introduced today.

Connection to Other Activities and the Whole Day

This activity comes near the end of the day and is a companion of the Beliefs activity at the beginning. The intent is that the present state of stakeholder beliefs from the lists generated earlier will provide a framework for generating likely concerns.

How To Carry Out the Activity

- **Grouping**
  Groups are within-area, as in the Beliefs Activity, so the discussion can focus specifically on the same locale.
- **Discussion Method**
  Each group considers two stakeholders. They refer to the list of beliefs of those stakeholders generated earlier, and generate a list of likely concerns the same stakeholders would have about changing high school algebra in their local schools.
- **Time**
  Allow 5 minutes to introduce the activity and for groups to discuss the concerns.
  Allow 10 minutes for group reports.

Materials

- **TM2a results** The chart paper lists of beliefs from the earlier Beliefs activity
- **TM2b** Concerns About Changing HS Algebra handout
- New chart paper and markers

Activity 7
NCTM Communication Process Standard

Overview and Rationale
This small group discussion is meant to examine the day's activities through the lens of the NCTM Communications Standard.

Connection to Other Activities and the Whole Day
This activity comes near the end of the day. The intent is to have participants recognize the many examples from the day's activities that relate to the NCTM Communications Standard.

How To Carry Out the Activity

- **Grouping**
  Groups can be the same as in the previous activity, or you can mix up the groups.
- **Discussion Method**
  Each group identifies examples from the day's activities that illustrate the important points in the Communications Standard, which are displayed on an overhead during the discussion. They can record their work on the Communications Standard handout. Some groups then report to the whole group.
- **Time**
  Allow 10 minutes for discussion and 5 minutes for group reports.

Materials

- **TM4b (repeat)** NCTM Communications Standard overhead and handout

TM-11 Every Student Counts - High School
Reading Assignment for Day 2

1. The algebraic focus for Day 1 was Functions.
   - Read the sub-section in the Algebra Standard for Grades 9 – 12 entitled "Understand patterns, relations, and functions" in *Principles and Standards for School Mathematics*, pages 297-300.
   - This reading describes the main algebraic ideas that were the focus of Day 1.
   - Reflection Question: How does the view of algebra and functions in this reading relate to the activities that you completed on Day 1?

2. The benefits and barriers to teaching mathematics through problem solving, the overall teaching focus for the ESC program, are discussed from different perspectives in the following readings.
   - Read Chapter 13, "Phasing Problem-Based Teaching into a Traditional Educational Environment" by Copes & Shager in *Teaching Mathematics through Problem Solving Grades 6 – 12*, pages 195 – 205.
   - As the title indicates, this reading addresses the challenges and benefits of a gradual introduction of teaching through problem solving into a traditional classroom setting.
   - Reflection Question: According to these authors, how can teaching through problem solving be phased into a traditional mathematics classroom?
   - Read Chapter 2, "What Research Says About the NCTM Standards" by Hiebert in *A Research Companion to Principles and Standards for School Mathematics*, pages 5 – 23. Copy of the chapter is in your ESC Notebook.
   - This chapter is a readable and complete summary of the research base for *Principles and Standards for School Mathematics*.
   - Reflection Question: According to research, how does what students often learn in traditional mathematics classes compare to what they learn in alternative (NCTM Standards-based) programs?

3. Reasoning and Proof will be the teaching theme for Day 2.
   - Read "Reasoning and Proof" for Grades Pre-K – 12 (pp. 122- 126) and for Grades 9 –12 (pp. 342-346) in *Principles and Standards for School Mathematics*.
   - Be prepared to analyze the analyze the Day 2 activities in terms of the four goals of the Reasoning and Proof Standard.
   - Reflection Question: According to these readings, what is the teacher's role in developing reasoning and proof in mathematics classes?